Statistical and Stochastic Problems in Ocean Modeling and Prediction

L. I. Piterbarg
University of Southern California
Center of Applied Mathematical Sciences
Kaprielian Hall, R108, 3620 Vermont Avenue
Los Angeles, CA 90089-2532
Phone (213) 740 2459, fax (213) 740 2424, e-mail piter@usc.edu

Award # N00014-99-1-0042
http://www.onr.navy.mil/sci_tech/ocean/onrpgahm.htm

LONG-TERM GOALS

My project addresses statistical and stochastic problems in the following fields: Lagrangian data assimilation (1), Lagrangian prediction (2), development and validation of Lagrangian stochastic models (LSM) (3). The long range scientific objectives of this study comprise rigorous determining limits of predictability for Lagrangian motion in semi-enclosed seas, littoral zones, and straits on time scales of days and weeks, elaborating concrete prediction schemes, developing optimal Lagrangian data assimilation algorithms, and identification of multi particle stochastic models aimed at incorporating them to ocean circulation models (OCM).

OBJECTIVES

The objectives for the third year of research were:

- Development and verification of estimation algorithms for Eulerian velocity field given Lagrangian data and distributed tracer observations based on the non-linear filtering approach.

- Construction and verification of data fusion procedures for computing surface velocities given a circulation model output, drifter data and observations of a continuously distributed tracer.

- Extension of the Lagrangian subgridscale model [1] to multi particle motion for improvement of Lagrangian transport prediction.

- Investigation of particle-pair statistics in the framework of a LSM with autocorrelated forcing.

APPROACH

I develop theoretical approaches to the Lagrangian prediction, Lagrangian data assimilation as well as to data fusion, in context of the theory of random processes and fields covered by stochastic partial differential equations. I design computational algorithms derived from the theoretical findings. During the last year I also made use of possibility theory and fuzzy logic to design procedures for optimal combining (fusion) data coming from different sources. A significant part of validating the algorithms is a testing them via stochastic simulations. Such an approach provides us with an accurate error analysis. Together with my collaborators from Rosenstiel School of Marine and Atmospheric Research

Report Documentation Page

Form Approved
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE **30 SEP 2007**	2. REPORT TYPE **Annual**	3. DATES COVERED **00-00-2007 to 00-00-2007**

4. TITLE AND SUBTITLE **Statistical And Stochastic Problems In Ocean Modeling And Prediction**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **University of Southern California,Center of Applied Mathematical Sciences,Kaprielian Hall, R108, 3620 Vermont Avenue,Los Angeles,CA,90089**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
code 1 only

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**8**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

(RSMAS), Consiglio Nazionale delle Ricerche (ISMAR, LaSpezia, Italy), Naval Research Laboratory (Stennis Space Center, Mississippi), ENEA (Rome, Italy), Koç University (Istanbul, Turkey) we implement the algorithms in concrete ocean models such as QG, MICOM , and NCOM as well as carry out statistical analysis of real data sets by means of new methods.

WORK COMPLETED

1. Assimilation and data fusion.

The problem of assimilation both drifter and image data in an Eulerian model is formulated as an optimal filtering problem. A quasi-optimal solution of the problem has been found for a wide class of stochastic models which are able to realistically mimic the upper ocean turbulence. A theoretical error analysis is developed for some important asymptotical cases. The approach requires a particular knowledge of the Eulerian velocity field statistics. For this purpose special methods were developed to estimate appropriate parameters from Lagrangian data.

For the same problem a different approach was developed and tested based on the possibility theory ideas, [2,3].

2. *Lagrangian prediction.*

A procedure was developed to reduce errors in the relative dispersion obtained from a numerical model by employing drifter statistics estimated from observational data sets. The method is based on the PI results regarding to the relative dispersion in multi particle Lagrangian stochastic models [4]. Earlier a similar approach resulted in improvement of modeling the absolute dispersion [1].

3. *Development of multi particle Lagrangian stochastic models.*

A new stochastic model for describing a particle pair has been formulated where the acceleration has a finite correlation time. Such an approach allows for consideration of the particle pair statistics as functions of both Kubo and Stokes numbers. A comprehensive analytical and Monte Carlo investigation of the Lyapunov exponent was carried out to establish the dependence of predictability and mixing properties on these two parameters.

RESULTS

1. The first principal result in the assimilation problem is derivation and verification of optimal filtering equations for the underlying Eulerian velocity given a model output $\mathbf{u}_m(t, \mathbf{x})$, observations of Lagrangian trajectories $\mathbf{r}_k(t)$\, k = 1,2,... driven by the real velocity field, and grid observations of a continuously distributed tracer $c(t,\mathbf{x})$ satisfying a standard advection-diffusion equation with a white noise forcing. For simplicity we show the filtering equations in the case of one drifter and tracer observations in a single point $\mathbf{x}$ only

$$\begin{aligned} d\widehat{\mathbf{u}} &= (-\widehat{\mathbf{u}}/\tau + \mathbf{F}(t,\mathbf{x}))dt + B(\mathbf{x},\mathbf{r}(t))B(\mathbf{r}(t),\mathbf{r}(t))^{-1}(d\mathbf{v} + (\mathbf{v}/\tau - \mathbf{F}(t,\mathbf{r}(t))dt) \\ &-\Gamma\nabla c R(\mathbf{x},\mathbf{x})^{-1}(dc + (\widehat{\mathbf{u}}\cdot\nabla c - D\nabla^2 c)dt) \end{aligned} \tag{1}$$

$$d\Gamma/dt = -2\Gamma/\tau + B(\mathbf{x},\mathbf{x}) - B(\mathbf{x},\mathbf{r}(t))B(\mathbf{r}(t),\mathbf{r}(t))^{-1}B(\mathbf{x},\mathbf{r}(t))^T - \Gamma\nabla c R(\mathbf{x},\mathbf{x})^{-1}\nabla c^T\Gamma^T \tag{2}$$

Here $\hat{u}$ is the velocity estimate, Γ its error matrix, $\mathbf{F}(t, \mathbf{x}) = \partial \mathbf{u}_m/\partial t - \mathbf{u}_m/\tau$, τ the Lagrangian correlation time, D diffusivity, $R(\boldsymbol{x},\boldsymbol{y})$ space covariance of the forcing and $B(\boldsymbol{x},\boldsymbol{y})$ the space covariance of velocity fluctuations. In addition we have developed a procedure of estimating the crucial parameters of the covariance $B(\boldsymbol{x},\boldsymbol{y})$ from Lagrangian data.

Essentially, (1-2) is a Kalman filter written in the continuous form which has been applied before for data assimilation (e.g. [5,6]). *The news here is that for a formulated general enough model it gives an exact solution rather than an approximation based on a linearized version of a sophisticated hydrodynamics model.* We have tested the discretized version of this algorithm modified for multiple drifters in the case with no tracer observations. Comparison of the error with the simplest interpolation algorithm (SA), [7], for different values of the space correlation radius, shows that the performance of the new method is clearly better, Fig. 1

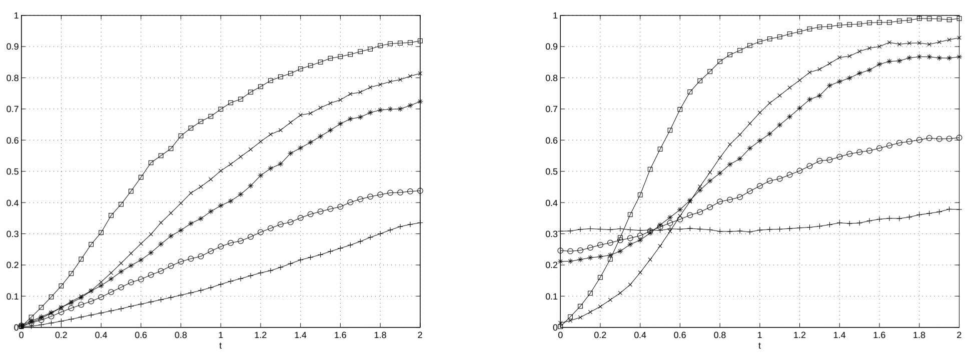

Figure 1. Left panel: velocity estimation error for algorithm (1,2) vs time (days) at different values of space correlation radius R=2 km (square), 4 km (cross), 6 km (star), 8 km (circle), 10 km (plus). Right panel: same for SA}

We also have tested another approach to the same problem based on the possibility theory [1,2]. In the simplest case of an isotropic confidence region for a circulation model output and tracer observations only , the fusion formulas are as follows

$$\widehat{u} = u_m - \frac{A(Au_m + Bv_m + E)}{A^2 + B^2}, \quad \widehat{v} = v_m - \frac{B(Au_m + Bv_m + E)}{A^2 + B^2} \qquad (3)$$

where u_m, v_m are the velocity estimates obtained from the circulation model (possibly with drifter data assimilation) and $A = \partial c/\partial x$, $B = \partial c/\partial y$, $E = \partial c/\partial t$ are taken at the underlined point.

This method allows a rigorous error analysis. One of our findings is that if $\sigma^2_{model} = \langle |\mathbf{u}_m - \mathbf{u}|^2 \rangle$ is small enough where $\mathbf{u}$ is the real velocity and the angle brackets mean the space averaging, then σ^2_{fusion} $\langle |\hat{u}_m - \mathbf{u}|^2 \rangle$ is twice as less as σ^2_{model}.

To test the algorithm we used an idealized velocity field in a rectangle region which is a superposition of the gyre with the stream function $\psi = \omega(x^2 + y^2)$ and periodic ageostrophic perturbations. It is assumed that some particular model is not able to capture those perturbations and its output coincides with ψ. Next a tracer is spread over the region by the real velocity. The estimation result for the zonal component after one day is shown in Fig. 2. The resulting σ^2_{fusion} = 7% is in full agreement with the

theoretical error analysis since we have taken σ^2_{model} = 10%, but what is more important that the method is capable to capture the qualitative structure of the current.

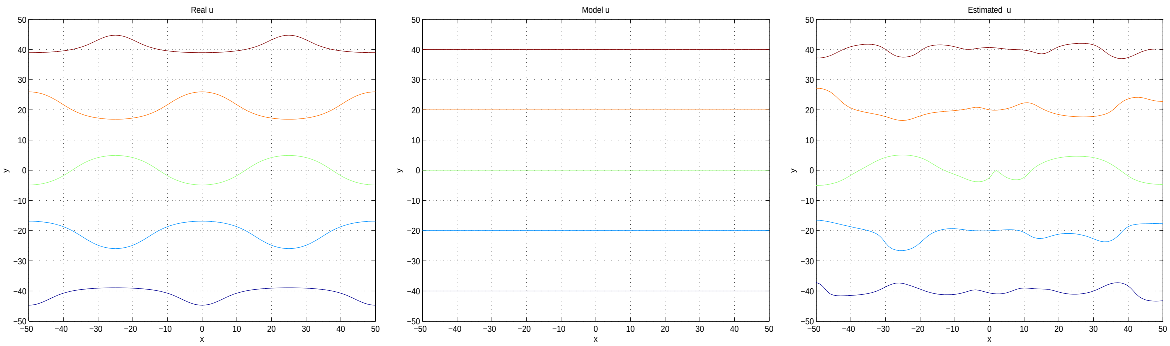

Figure 2. Real u (left), model u (center) and the estimated velocity by fusion of model output and tracer observations via (3) (right)

2. The most important finding in our Lagrangian prediction study is a computationally efficient algorithm of correcting model generated trajectories in a way such that the resulting relative dispersion is close to that of estimated from observations. For simplicity we show only the corresponding equation for the zonal component of Lagrangian velocity. Namely, let $u_r(t,\mathbf{r})$ and $u_m(t,\mathbf{r})$ be a real and model velocities described for any label $\mathbf{r}$ by the Langevin equation with different parameters, (σ_r, τ_r) and (σ_m, τ_m) respectively. We have proven that under certain realistic conditions the corrected velocity $u = u_m + \eta$ generates the same absolute and relative dispersion if the missing component η satisfies the following stochastic differential equation

$$\frac{d\eta}{dt} = a\frac{d\tilde{u}_m}{dt} + b\tilde{u}_m + c\eta$$

where the coefficients a, b, c are expressed in terms of (σ_r, τ_r) and (σ_m, τ_m) exactly as in [1] and $\tilde{u}_m = Lu_m$ where A is a linear operator applied to the label. L depends on some parameters characterizing the space covariance structure of the real and model velocity fields. In particular if the parameters are close for the real and model velocity fields, then L is the identical operator and the method reduces to the known one [1], where only one-particle statistics are intended to match. The mentioned parameters such as the space correlation scale and the energy slope on large scales are efficiently estimated from Lagrangian data [8].

3. In the new suggested stochastic particle-pair model, the dependence on Stokes (ε) and Kubo (h) numbers of the Lyapunov exponent (LE) of particle trajectories reveals the presence of a region in parameter space (ε, h) where LE changes sign, thus signaling the transition, Fig. 3.

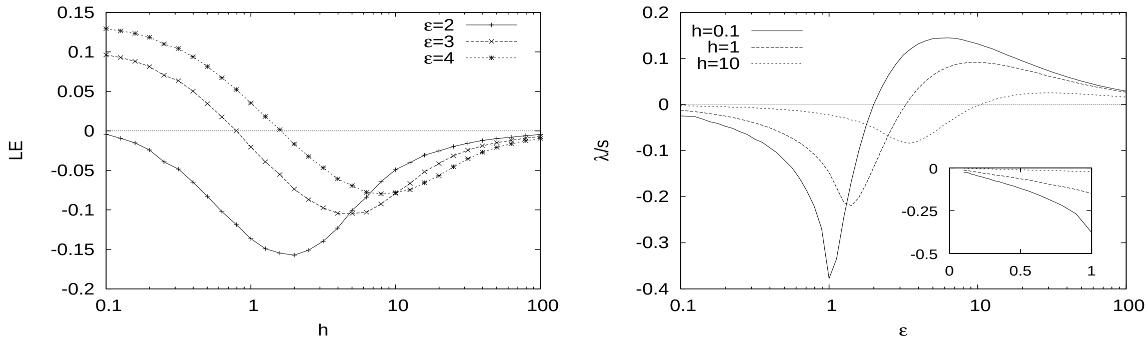

Figure 3. Left panel: dependence of LE on Kubo number (h) for different Stokes numbers (ε). Right panel: dependence of LE on Stokes numbers (ε) for different Kubo number (h)

In particular, for large Stokes number, a regime characterized by the formation of shocks, we found a chaotic region in parameter space (ε, h), where LE becomes positive. Inertia is therefore responsible for a transition from a strong clustering regime, originated by the compressible nature of the flow, to a chaotic regime. The latter is observed in a range of h such that the time correlation of fluid gradients is long enough to provide substantial stretching, but not too long to get particles trapped in compressing regions.

IMPACT/APPLICATIONS

1. The suggested data fusion procedures for estimating Eulerian velocity given Lagrangian observations , images, and a model output could be a supplement to existing assimilation algorithms explicitly involving OGCM.

2. The suggested procedure of adding a missing component to Lagrangian velocities coming from OGCM can potentially improve their performance for particle pair statistics such as the relative dispersion, Lyapunov exponent and finite time Lyapunov exponent.

3. The developed theory of the Lyapunov exponent along with estimates of the Stokes and Kubo numbers for the oceanic conditions would lead to classification different regions with respect to the mixing rate and the accuracy of prediction of the Lagrangian trajectories.

TRANSITIONS

The developed trajectory correction algorithm was used in RSMAS to test it in NCOM circulation model. Its new particle pair version is planned to implement in an OGCM by the same RSMAS group.

RELATED PROJECTS

1. "Predictability of Particle Trajectories in the Ocean", ONR, PI T.Ozgokmen , RSMAS, N00014-05-1-0095

2.   "Lagrangian turbulence and transport in semi-enclosed basins and coastal regions", ONR, PI A Griffa, RSMAS, N00014-05-1-0094

REFERENCES

1. A. C. Haza, L. I. Piterbarg, P. Martin, T. M. Ozgokmen, and A. Griffa, (2007), A Lagrangian subgridscale model for particle transport improvement and application in the Adriatic Sea using the Navy Coastal Ocean Model, *Ocean Modelling*, v.17, 68-91

2. D. Dubious and H. Prade, (1986), Possibility theory, Plenum Press, New York and London,

3. D. Dubois, H. Prode, and R. R. Yager, Fuzzy Information Engineering, 1997, John Wiley & Sons, Inc.

4. L. I. Piterbarg, (2005), Relative dispersion in 2D stochastic flows, *J. of Turbulence*, v.6, n.4, 1-19.

5. Carter, E. F., (1989), Assimilation of Lagrangian data into a numerical model. *Dyn. Atmos. Oceans*, **13**, 335-348.

6. Ide, K., L. Kuznetsov and C. K. R. T. Jones, (2002), Lagrangian data assimilation for point-vortex system *J. Turbulence*, **3**, 053.

7. Molcard A., L. Piterbarg, A. Griffa, T. Ozgokmen, and A. Mariano., (2003): Assimilation of Lagrangian data in Eulerian models., J. Geophys. Res., v.108, n. C3, 3056.

8. L. I. Piterbarg, (2006), Parameter estimation in multi particle Lagrangian stochastic models, *Monte Carlo Methods and Applications*, v.12, n.5-6, 477-494.

PUBLICATIONS

M. I. Caglar, T. M. Ozgokmen, and L .I. Piterbarg, (2006), Parameterization of submeso-scale eddy-rich flows using a stochastic velocity model, *J. Atmospheric and Oceanic Technology*, v.33, n.12, 1745-1758.

L. I. Piterbarg, (2006), Parameter estimation in multi particle Lagrangian stochastic models, *Monte Carlo Methods and Applications*, v.12, n.5-6, 477-494.

A. C. Haza, L. I. Piterbarg, P. Martin, T. M. Ozgokmen, and A. Griffa, (2007), A Lagrangian subgridscale model for particle transport improvement and application in the Adriatic Sea using the Navy Coastal Ocean Model, *Ocean Modelling*, v.17, 68-91.

L. I. Piterbarg, (2007), Optimal estimation of Eulerian velocity field given Lagrangian observations, *Applied Mathematical Modelling* in press.

G. Falkovich, S. Musacchio, L. Piterbarg, M. Vucelja, (2007), Inertial particles driven by a telegraph noise, *Physical Review E*, v.76,
n.2, 1-8.

L. I. Piterbarg, T. Ozgokmen, A. Griffa , and A. Mariano (2007), Predictability of Lagrangian trajectories, in *Lagrangian Analysis and Prediction of Coastal and Ocean Dynamics*, ed. A. Griffa, D. Kirwan, A. Mariano, T. Ozgokmen, and T. Rossby, Cambridge University Press, 2007, 136-171.

A. Molcard, T. Ozgokmen, A. Griffa, L.I. Piterbarg, and M. Chin (2007), Lagrangian data assimilation in ocean general circulation models, in *Lagrangian Analysis and Prediction of Coastal and Ocean Dynamics*, ed. A. Griffa, D. Kirwan, A. Mariano, T. Ozgokmen, and T. Rossby, Cambridge University Press, 2007, 171-203.

L. I. Piterbarg and M. Gaglar (2007), Research Trends in Stochastic Flows Modeling the Upper Ocean Turbulence, In *Applied Mathematical Modeling Research Trends*, ed. F. Columbus, Nova Science Publishers, Inc., accepted.